Abstract

By a derivation, we mean an additive mapping $d$ on a ring $R$ such that $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. Derivations on rings play a significant role in ring theory. In 1957, Posner proved a milestone result related to derivations on rings, and investigated the structure of rings and nature of mappings. A number of authors have extended Posner’s result in various directions such as generalized derivation, Jordan derivation, $(\alpha, \beta)$-derivation, generalized $(\alpha, \beta)$-derivation, multiplicative derivation, multiplicative generalized derivation etc., on appropriate subsets such as ideal, left ideal, right ideal, Lie ideal, Jordan ideal, etc. In particular, motivated by the fact that a non central Lie ideal contains all commutators of a nonzero ideal of a ring with char($R$) $\neq 2$, Lee et al. generalized the Posner’s result on multilinear polynomials. Afterwards, several authors studied different identities involving various mappings on multilinear polynomials. In this thesis, we study identities involving generalized derivations with multilinear polynomials on prime rings, and obtain the structure of mappings and rings.

There is a vast literature which studies identities on different derivations involving annihilating conditions. The seminal result in this direction was given by Posner who proved that if $a \in R$ and $d$ is a derivation on prime ring $R$ such that $ad(x) = 0$ for all $x \in R$, then either $a = 0$ or $d = 0$. Many authors extended this result in various directions. In this thesis, we study the identities involving generalized derivation on prime ring $R$ with annihilating condition and obtain the commutativity of $R$. We also investigate annihilating condition of an identity involving automorphism on prime ring.

Jacobson in his book “Structure of Rings” has introduced the notion of $(s_1, s_2)$- derivation which was later more commonly known as $(\alpha, \beta)$-derivation. In this direction, in this thesis we study the connection between the nature of generalized $(\alpha, \beta)$-derivation on appropriate subsets of prime ring $R$ and commutativity of $R$. If we remove the additivity property of generalized $(\alpha, \beta)$-derivation $G$, then it is called multiplicative generalized $(\alpha, \beta)$-derivation. More-
over, if we drop all restrictions on its associated map $d$ as well, that is, if $d$ is any map, then $G$ is called multiplicative (generalized)-$(\alpha, \beta)$-derivation. In this thesis, we examine those conditions on a ring $R$ for which multiplicative generalized $(\alpha, \beta)$-derivation on $R$ becomes additive. In this sequence, we also study the identities involving multiplicative (generalized)-$(\alpha, \beta)$-derivation on left ideal of a prime ring and derive some important results.